

Example of the Improved Euler Method

IVP: $\frac{dy}{dx} = 2xy$

$y(0) = 2$

step size: $h = 0.1$

x	y	h	k1	x+h	y+k1	k2	Δy
0	2	0.1	0	0.1	2	0.04	0.02
0.1	2.02	0.1	0.0404	0.2	2.0604	0.082416	0.061408
0.2	2.081408	0.1	0.083256	0.3	2.164664	0.12988	0.106568
0.3	2.187976	0.1	0.131279	0.4	2.319255	0.18554	0.158409
0.4	2.346386	0.1	0.187711	0.5	2.534096	0.25341	0.22056
0.5	2.566946	0.1	0.256695	0.6	2.82364	0.338837	0.297766
0.6	2.864712	0.1	0.343765	0.7	3.208477	0.449187	0.396476
0.7	3.261188	0.1	0.456566	0.8	3.717754	0.594841	0.525703
0.8	3.786891	0.1	0.605903	0.9	4.392794	0.790703	0.698303
0.9	4.485194	0.1	0.807335	1	5.292529	1.058506	0.93292
1	5.418114						

Comparison of the results obtained from the Improved Euler Method by using various step sizes.

IVP : $y' = 2xy$, $y(0) = 2$

x	h = 0.1 y	h = 0.05 y	h = 0.01 y	h = 0.001 y	Correct y *
0	2	2	2	2	2
0.1	2.020000	2.020088	2.020100	2.020100	2.020100
0.2	2.081408	2.081593	2.081621	2.081622	2.081622
0.3	2.187976	2.188294	2.188348	2.188349	2.188349
0.4	2.346386	2.346916	2.347019	2.347022	2.347022
0.5	2.566946	2.567843	2.568045	2.568051	2.568051
0.6	2.864712	2.866257	2.866646	2.866659	2.866659
0.7	3.261188	3.263870	3.264605	3.264632	3.264632
0.8	3.786891	3.791548	3.792909	3.792961	3.792962
0.9	4.485194	4.493249	4.495717	4.495815	4.496816
1	5.418114	5.431980	5.436382	5.436562	5.436564

* $y = 2e^{x^2}$